Dyons in QCD: Confinement and Chiral Symmetry Breaking*

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Abstract

Dyonic classical solutions of $SU(2)$ gluodynamics are discussed. Exact form of dyonic solutions in different gauges is presented and the nontrivial problem of composition of the dilute gas of dyons is settled.

Classical interaction between (anti)dyons is considered both analytically and numerically. Confinement in the dyonic gas is discussed in connection with the topological properties of individual dyon solution.

Fermionic zero modes of dyonic are displayed and the chiral symmetry breaking in the dyonic gas is demonstrated.

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1 Introduction

The QCD vacuum is known to possess properties of confinement and chiral symmetry breaking (CSB). In absence of dynamical quarks (quenched approximation) confinement is characterized by the area law of Wilson loop or zero average of Polyakov line, while CSB is connected to nonzero values of chiral quark condensate. It was found in lattice calculations [1] that both properties disappear at the same temperature T_c, while a part of confining configurations survive for $T > T_c$, ensuring area law for spacial Wilson loops [2] ("magnetic confinement" [3]).

In addition, the QCD vacuum is characterized by the topological susceptibility χ and nonperturbative energy density or gluonic condensate [4], both quantities imply (in terms of gas of topological charges, like instantons) a density of approximately 1 top charge per $1fm^4$ [5,6].

By now there is no model of QCD vacuum with properties of confinement and CSB, based directly on the QCD Lagrangian.

The most elaborated model is the instanton gas or liquid model (IM) [6], which ensures CSB but lacks confinement [7]. Even so the instanton model shows rather realistic features for hadron correlators [8] demonstrating that CSB is already very important property for correlators.

The same can be said about the Nambu–Jona-Lasinio model (NJL) [9] (not directly connected to QCD Lagrangian) where confinement is also absent. Therefore in both IM and NJL hadrons can dissociate into quarks.

Thus it is an urgent need to look for more realistic model of QCD vacuum which obeys both basic properties: CSB and confinement. The latter is associated widely with monopole-like degrees of freedom [10], which may be of purely quantum or quasiclassical character. In the latter case one should look for classical solutions of Yang–Mills theory with monopole – like fields. These solutions are known for a long time [11]. Such solution can be obtained from the multiinstanton solution in the so-called 'tHooft's ansatz by the singular gauge tranformation [11]. In the general case of finite–action multiinstanton solutions the form of fields and generalized gauge transformation were found in [12].

The solutions have both color–electric and color–magnetic fields and we therefore shall call them dyons.

The dilute dyonic gas has been suggested some time ago as a model of QCD vacuum and some simple estimates of Wilson loop has been done for

dyons of finite time extension [12], demonstrating nonzero string tension. Recently the interest for the dyonic solutions has revived. In particular lattice studies of a classical and quantum field of a dyon have been done and a qualitative quasiabelian picture of confinement due to dyons was suggested [13].

Meanwhile the CSB properties of dyonic gas has been studied [14]. It appears that each dyon has (infinitely many) zero fermionic modes [15] and therefore CSB may occur through the same or similar mechanism as in the IM, where each instanton has a zero fermionic mode and the gas of instantons create collectively chiral quark mass and quark condensate [16]. It was shown in [14] that this indeed happens, and the values of chiral mass and condensates depend on density of dyons in the gas.

Recently in an interesting series of papers [17] the properties of the solvable $N = 2$ SUSY, $4d$ model containing gluons, Majorana fermions and an adjoint Higgs field were studied. It was demonstrated [17] that there is a confining phase in the model with confinement driven by the condensation of dyons and magnetic monopoles. With that the dual-Meissner effect as confinement mechanism [10] obtains an independent support, and also the dyonic gas model gets an additional impetus.

It is a purpose of this lecture to consider general properties of the dyon gas from point of view of confinement and CSB and to obtain estimates of the corresponding parameters. To this end we refine our ansatz of dyonic gas done 10 years ago [12] and try to obtain a self-consistent picture adjusting two parameters of the model: average size of dyon ρ and average distance between them, R. With some choice of ρ and R we show that one can reasonably reproduce values of string tension σ, quark condensate $< \bar{q}q >$, gluonic condensate $G_2 = \frac{\alpha_s}{\pi} < (F^a_{\mu\nu})^2 >$, topological susceptibility χ. Moreover, one can calculate all field correlators and compare with existing Monte-Carlo data.

As additional check of the model we study a possible scenario of temperature phase transition and discuss nonperturbative physics at $T > T_c$.

The lecture is organized as follows. In chapter 2 we define a single dyon solution, make a gas of dyons in chapter 3 and calculate Wilson loop average for a single dyon and dyon gas in chapter 4. We compare this result with the case of instanton and instantonic gas and demonstrate the reason why dyons confine while instantons do not. In computing string tension an important role is played by dyon-dyon and dyon-antidyon correlations and screening

length of dyon magnetic and electric charge, which we specifically study.

In chapter 5 we calculate CSB for dyonic gas using resent results [14], and expressing effect in terms of ρ and R.

In conclusion the deconfinement scenario for dyonic gas is discussed and some estimates for the dyonic gas are presented.

2 Properties of dyonic solutions

We remind the classical Yang–Mills solution in the so–called 'tHooft ansatz

$$A^a_\mu = -\frac{1}{g}\bar{\eta}^a_{\mu\nu}\partial_\nu lnW \tag{2.1}$$

where $\bar{\eta}^a_{\mu\nu}$ is 'tHooft symbol

$$\bar{\eta}^a_{\mu\nu} = e_{a\mu\nu},\ \mu,\nu = 1,2,3 \text{ or } \delta_{a\nu},\ \mu = 4, \text{or} - \delta_{a\mu},\ \nu = 4 \tag{2.2}$$

and

$$W = 1 + \sum_{i=1}^{N} \frac{\rho_i^2}{(x-x_i)^2} \tag{2.3}$$

ρ_i and $x_i,\ i = 1, \ldots N$ are real; for finite $\rho_i = \rho$ one has the Harrington – Shepard solutions [11], but we are interested in the limit $\rho_i = \rho \to \infty$ and x_i lying equidistantly along the straight line

$$x_k = \vec{r},\quad kb,\quad k = 0,\quad \pm 1,\quad \pm 2,\quad \ldots \pm N_1, \tag{2.4}$$

In most cases the limit of $N_1 \to \infty$ will be considered.

For the choice $\rho \to \infty$ one has

$$W(r,t) = \sum_{k=0,\pm1,\ldots} \frac{1}{r^2 + (t-kb)^2} \tag{2.5}$$

and A^a_μ due to (2.1) is in general periodic in time t. One can make a gauge transformation $\tilde{A}_\mu = \tilde{A}^a_\mu \frac{t_a}{2} = U^+(A_\mu + \frac{i}{g}\partial_\mu)U$, with $U = exp(i\frac{\vec{\tau}\vec{n}}{2}\theta)$, such that $\tilde{A}_\mu$ is time–independent for $N_1 \to \infty$ [11,12]. In this limit one has

$$\tilde{A}_{ia} = f(r) e_{iba} n_b \tag{2.6}$$

$$\tilde{A}_{4a} = \varphi(r) n_a \tag{2.7}$$

with

$$f(r) = \frac{1}{gr}(1 - \frac{\gamma r}{sh\gamma r}) \tag{2.8}$$

$$\varphi(r) = \frac{1}{gr}(\gamma r cth\gamma r - 1), \quad \gamma = \frac{2\pi}{b} \tag{2.9}$$

We shall call (2.6)-(2.7) the Rossi solution [11] or the dyon solution since it has both electric and magnetic fields; it is clear that Rossi solution belongs to the class of Prasad–Sommerfield solutions [11].

One can easily find color–electric and color–magnetic fields, $E_{ka} = -B_{ka}$

$$B_{ka} = \delta_{ak}(-f' - f/r) + n_a n_k (f' - f/r + gf^2) \tag{2.10}$$

The field (2.9) contains both long-range and short–range parts, indeed for large r one has $f(r) \sim 1/gr$ and $B_{ka} \sim -\frac{n_a n_k}{gr^2}$. To make separation more clear let us go from the hedgehog gauge(2.5-2.6) to the quasiabelian (or unitary) gauge, where the long–range part of B_k is Abelian. This gauge rotation is given by the orthogonal matrix [18]

$$0_{ik} = cos\theta \delta_{ik} + (1 - cos\theta)\nu_i \nu_k + sin\theta e_{ikl}\nu_l \tag{2.11}$$

where we have introduced unit vector $\vec{e}$ and defined

$$cos\theta \equiv \vec{e}\vec{n}; \quad \nu_i sin\theta = -e_{imn} e_m n_n \tag{2.12}$$

One also has property

$$0_{ik} n_i = e_k \tag{2.13}$$

Therefore the gauge transformed $A'_{\mu a}$,

$$A'_{\mu a} = \tilde{A}_{\mu b} \cdot 0_{ba} - \frac{1}{2g} 0_{lb} \partial_\mu 0_{lc} e_a bc \tag{2.14}$$

yields

$$A'_{4a} = \varphi(r) e_a \tag{2.15}$$

and

$$B'_{ka} = (-2f/r + gf^2) n_k e_a + (-f' - f/r)[cos\theta \delta_{ka} + (1 - cos\theta)\nu_k \nu_a - e_k n_a] \tag{2.16}$$

Now we notice that the long–range part – the first term on the r.h.s. of (2.16) – has a fixed color direction; by choosing $\vec{e}$ along the third axis, we have

$$- E'_k = B'_k(r \to \infty) \sim -\frac{1}{gr^2} \cdot n_k \cdot \frac{\tau_3}{2} \tag{2.17}$$

of course this transformation from (2.10) to (2.16) not defined at $\theta = \pi$. (Dirac string). We note also, that the long-range part of B_k and E_k can be written as a derivate

$$\vec{B}' = \nabla\Phi\frac{\tau_3}{2} + \text{ short-range terms} \tag{2.18}$$

with

$$\Phi = \int_0^r (-2f/r' + gf^2)dr' \tag{2.19}$$

The total action of dyon is proportional to its time extension

$$S = \frac{1}{2}\int d^3\vec{r}\int_0^T dt(B_{ak}^2 + E_{ak}^2) = \frac{8\pi^2}{g^2 b}T \tag{2.20}$$

It can be considered as a string of instantons of infinite radius [12], and for N centers in the string (the "N-string") one has

$$S(N) = \frac{8\pi^2}{g^2}(N-1) \tag{2.21}$$

The total number of parameters for the case when we allow the centers to move and have finite radii ρ_i is equal to $5N-1, (N>3)$ [19] (plus 3 overall color orientations).

Thus the enthropy of the N–string is smaller than that of N independent instantons – in the latter case the total number of parameters is $8N$ due to independent color orientation of each instanton. This is the price one should pay for a new property of dyon: its coherent field is able to confine, as we shall demonstrate later.

Now we turn to the quantum corrections around the dyon. There is a literature on the subject [20], but we shall use a simple method a lá Polyakov to obtain the effective action of dyon due to quantum corrections around the dyonic classical solution.

One can write the effective action S_{eff} as [21]

$$S_{eff} = \frac{1}{4}\int \frac{d^4q}{(2\pi)^4}(\frac{1}{g_0^2} + \pi(q^2))F^a_{\mu\nu}(q)F^a_{\mu\nu}(-q) \tag{2.22}$$

where the gluon plus ghost self–energy part $\pi(q^2)$ is

$$\pi(q^2) = -\frac{11}{3}N_c\frac{1}{16\pi^2}ln\frac{\Lambda_0^2}{q^2} \tag{2.23}$$

and one can introduce the renormalized charge

$$\frac{1}{g^2(q)} = \frac{1}{g_0^2} - \frac{11}{3}N_c\frac{1}{16\pi^2}ln\frac{\Lambda_0^2}{q^2} = \frac{b_0}{16\pi^2}ln\frac{q^2}{\Lambda^2} \tag{2.24}$$

For a long (anti)dyon $N \gg 1$) one can write

$$\pm E_{ka} = B_{ka} = \zeta(q_0)b_{ka}(|\vec{q}|) \tag{2.25}$$

with $\zeta(q_0) = T\frac{sin(q_0T/2)}{(q_0T/2)}$. Introducing (2.24) in (2.21) one obtains

$$S_{eff} \approx \frac{T}{b}\frac{8\pi^2}{g^2(\tilde{q})}, \tag{2.26}$$

where $g^2(\tilde{q})$ is given in (2.24) and $\tilde{q}$ is obtained from the integral $\int b^2_{ka}\frac{(q)}{(2\pi)^3}\frac{d^3q}{g^2(q)} \approx \frac{1}{g^2(\tilde{q})}\frac{16\pi^2}{b}$

$$\tilde{q} \approx \frac{2\gamma}{\pi} \approx \frac{4}{b} \tag{2.27}$$

The form (2.23) is valid when $q \gg \Lambda$; for small q/Λ one should replace (2.23) by the expression which takes into account confining configurations in the vacuum [21,22] – dyonic gas in our case – and one has approximately

$$\frac{1}{g^2(q)} \approx \frac{b_0}{16\pi^2}ln\frac{q^2 + m_0^2}{\Lambda^2}, \quad m_0^2 \approx 2\pi\sigma \tag{2.28}$$

3 Dyonic gas

In this and following chapters the results of ref. [23] are largely used. Consider a system of several dyons and antidyons. We have several options for the composition of the system.

(1) Dyonic gas as a simple superposition ansatz (similar to the instanton gas ansatz [6])

$$A_\mu(x) = \sum_{i=1}^{N_+} A_\mu^{+(i)}(x) + \sum_{i=1}^{N_-} A_\mu^{-(i)}(x) \tag{3.1}$$

where N_+ and N_- are numbers of dyons and antidyons respectively. Each individual vector potential $A_\mu^{(i)}$ (with superscript + for dyons and - for antidyons) can be characterized by a $4d$ vector $R^{(i)}$ and $0(4)$ unit vector $\omega^{(i)}$, defining direction of the straight line passing through $R^{(i)}$ and centers kb in (2.5), moreover there is an overall color orientation $\Omega^{(i)}$, so that

$$A_\mu^{(i)}(x) = \Omega_i^+ (L\tilde{A})_\mu(r,t)\Omega_i \tag{3.2}$$

where $\tilde{A}_\mu(r,t)$ is the solution defined in chapter 2 and

$$r = [(x - R^{(i)})^2 - ((x-R)\cdot\omega^{(i)}]^{1/2} \ , \tag{3.3}$$

$$t = (x-R)\cdot\omega^{(i)} \ , \tag{3.4}$$

$L_{\mu\nu}(\omega)$ is the $0(4)$ rotation matrix, corresponding to ω.

Thus the overall vector potential $A_\mu(x)$ in (3.1) depends on the set $\{\Omega_i, R^{(i)}, \omega^{(i)}\},\ i = 1, ..., N_+ + N_-$ and the total stochastic ensemble contains A_μ with all possible values of this set.

For the case of zero temperature, QCD vacuum should be $0(4)$ invariant in the sense, that every observable K is an average over stochastic ensemble of the operator $K(A)$

$$K =< K(A) >_{\Omega,R,\omega} \tag{3.5}$$

where the weight of averaging is Poincare–invariant with respect to R, ω and integration $d\Omega$ is with the usual Haar measure.

For nonzero temperature there appears a preferred direction in $4d$.

The relative simplicity of the ansatz (3.1) has a serious drawback –as also in the case of instanton gas – $A_\mu(x)$ is not a classical solution. As a consequence even two dyons has an interaction energy (action), and there are some additional divergencies, which we now discuss.

First of all one notices, that the action of the single dyon of fixed length L is finite together with quantum corrections and proportional to L. This is because $B_{ka} = \pm E_{ka} \sim \frac{1}{r^2}, \ \ r \to \infty$. For a pair of dyons situation is different. Let us consider the simplest case of dd or $d\bar{d}$ system with dyons in the same gauge (2.6-2.7), same $\omega^{(1)} = \omega^{(2)}$ – two static dyons.

$$A_{ia} = \frac{f(r_1)}{r_1} e_{iba} r_{1b} + \frac{f(r_2)}{r_2} e_{iba} r_{2b} \tag{3.6}$$

$$A^{\pm}_{4a} = \frac{\varphi(r_1)}{r_1} r_{1a} \pm \frac{\varphi(r_2)}{r_2} r_{2a} \tag{3.7}$$

and $\vec{r}_i = \vec{r} - \vec{R}^{(i)}, \ \ \vec{R}^{(i)}$ is position of a dyon.

For color magnetic and colorelectric field one has at large distances $r^{(i)} \gg \gamma^{-1}$

$$F_{12} = \frac{h_1(\vec{r}_1\vec{\tau})}{2gr_1^4} + \frac{h_2(\vec{r}_2\vec{\tau})}{2gr_2^4} - \frac{h_1\vec{\tau}\vec{r}_2 + h_2\vec{\tau}\vec{r}_1}{2gr_1^2r_2^2} \tag{3.8}$$

$$E_{ia} = E^{(1)}_{ia} + E^{(2)}_{ia} + E^{(12)}_{ia}, h_i \equiv R^{(i)}_3 \tag{3.9}$$

where $E^{(1)}_{ia}$ and $E^{(2)}_{ia}$ is given in (2.10) and the interaction term $E^{(12)}_{ia}$ is

$$E^{(12)}_{ia} = \frac{\gamma}{g}[-\delta_{ia}\frac{(\vec{r}_1\vec{r}_2)}{r_1r_2}(\frac{1}{r_2} \pm \frac{1}{r_1}) + \frac{r_{1i}r_{2a}}{r_1r_2^2} \pm \frac{r_{1a}r_{2i}}{r_1^2r_2}] \tag{3.10}$$

It is clearly seen in (3.10) that the colorelectric contribution to the action

$$S = \frac{1}{2}\int (E^2_{ia} + B^2_{ia}) d^3\vec{r}dt \tag{3.11}$$

is diverging at large r for the system of two dyons, since $E^{(12)}_{ia}(dd) \sim 0(\frac{\gamma}{r}), \ \ r \to \infty$; while for $d\bar{d}$ system one has

$$E_{ia}(d\bar{d}) \sim 0(\frac{\gamma R}{r^2}), \quad r \to \infty \ , \quad R = |\vec{R}^{(1)} - \vec{R}^{(2)}| \tag{3.12}$$

and the action is converging. From this it follows that it is impossible to have thermodynamic limit for $N_+ \neq N_-$ (as well as for the Coulombic gas with nonzero net charge, here situation is similar).

For the system of dyons and antidyons one can write for large distances

$$B_{3a} = \sum_{i=1}^{N} B^{(i)}_{3a} - \sum_{i\neq j=1}^{N} \frac{h_i r_{ja}}{2gr_i^2r_j^2} \ , \quad h_i \equiv R^{(i)}_3 \tag{3.13}$$

$$E_{ka} = \sum_{i=1}^{N} E_{ka}^{(i)} + \frac{1}{g}\left\{ -\delta_{ka} \sum_{i \neq j=1}^{N} \frac{\vec{r}_i \vec{r}_j}{r_i r_j} (\frac{\gamma_i}{r_j} + \frac{\gamma_j}{r_i}) + \right. \tag{3.14}$$

$$\left. \frac{r_{ik} r_{ja} \gamma_i}{r_i r_j^2} + r_{ia} r_{jk} \frac{\gamma_j}{r_j^2 r_j} \right\}$$

where $\gamma_i = +\gamma$ and $-\gamma$ for dyons and antidyons respectively. One can see that the only dangerous term in (3.14) is $0(\frac{\gamma}{r})$ and it vanishes at large r if $N_+ = N_-$, so that $\sum_{i=1}^{N} \gamma_i = 0$. Hence behaviour at large r for $N_+ = N_-$ is $B_{ka} \sim E_{ka} \sim 0(1/r^2)$.

Let us now calculate action of $d\bar{d}$ system, eq. (3.11) as a function of dastance R_{12} between d and $\bar{d}$. The leading term comes from $(E_{ia}^{(12)})^2$ and has the form

$$S_{int} \sim \frac{1}{2} \int (E_{ia}^{(12)})^2 d^3\vec{r} dt \sim T\gamma^2 R_{12} + 0(T\gamma) + ... \tag{3.15}$$

One observes in (3.15) linear confinement between d and $\bar{d}$, $V_{int} = \frac{S_{int}}{t} \sim \gamma^2 R_{12}$.

Even more striking is the behaviour of the $d^2\bar{d}^2$ system. One can compute total action of the system and $S_{int} = S_{total} - \sum_{i=1}^{4} S_i$.

S_{int} depends on the configuration of the $d^2\bar{d}^2$ system when two d and two $\bar{d}$ are relatively close, while distance between d^2 and $\bar{d}^2$, $R(d^2 - \bar{d}^2)$, is large, one again obtains linear confinement

$$S_{int} \sim T\gamma^2 R(d^2 - \bar{d}^2) \tag{3.16}$$

For configuration of a "molecule" consisting of two atoms $d\bar{d}$ with large $R(d\bar{d} - d\bar{d})$ one gets instead

$$S_{int} \sim T\gamma^2 \frac{(r(d\bar{d}))^4}{R^3(d\bar{d} - d\bar{d})}, \quad r(d\bar{d}) \sim \gamma^{-1} \tag{3.17}$$

This behaviour reminds of a color Van-der-Waals potential between color–neutral objects.

Thus a gas of static $(3d)$ dyons and antidyons with $N_+ = N_-$ degenerates at minimal action to the gas of $d\bar{d}$ atoms of size γ^{-1}. It is intersting to compute fields and action of the $d\bar{d}$ system, when distance between $d\bar{d}$, $r(d\bar{d})$

is not large. At $r(d\bar{d})=0$ the colorelectric field vanishes, $E_{ia}=0$; while colormagnetic field B_{ia} becomes short range, $B_{ia}(r(d\bar{d})=0)\sim exp(-\gamma r),\quad r\to\infty$. The total action is

$$S+T\frac{32\pi^2}{g^2b}\int_0^\infty\frac{dx}{sh^2x}[(1-xcthx)^2+2(1-x\cos echx)^2] \tag{3.18}$$

Numerical estimate yields

$$S(r(d\bar{d})=0)\cong S_0\frac{T}{b}1,1 \tag{3.19}$$

This should be compared with the action of each (anti)dyon, $S=S_0\frac{T}{b}$.

Thus one can see a strong attraction between d and $\bar{d}$ at small distances. The overal behaviour of the $d\bar{d}$ action numerically computed in [23] is shown in Fig.1. One can see both regimes (3.19) and (3.15). However, the lack of long–distance field, and zero topological charge of the $d\bar{d}$ atom makes it useless from the point of confinement, as we shall see in the next Section.

The situation with the dyonic–antidyonic gas in $4d$, i.e. when the (anti)dyon lines are have all directions, is different from the gas of static dyons. Indeed, let us take one dyon with the line along 4-th axis (i.e. static dyon) and antidyon with the line along the 3d axis. Assuming the superposition principle as in (3.6), one obtains for $E^{(12)}_{ia}$

$$E^{(12)}_{ia}\sim\gamma^2e_{abc}n^{(1)}_bn^{(2)}_c \tag{3.20}$$

It is seen from (3.20) that the action (3.11) diverges for any distance between d and $\bar{d}$, since $(E^{(12)}_{ia})^2$ is nonzero at large r.

Thus the $4d$ gas of d and $\bar{d}$ cannot exist in the ansatz (3.6-3.7) and one must choose another composition principle, which we consider next.

(2) The $4d$ model of dyonic gas

We are using again the superposition ansatz (3.1-3.4). At this point one must specify in which gauge $A^{(i)}_\mu(x)$ in (3.1) are summed up.

We have found only one gauge (modulo global rotations) where the action of the superposition (3.1) is finite. This is actually the singular gauge of the original 'tHooft ansatz (2.1),

$$A^{(i)}_{\mu a}=-\bar{\eta}_{a\mu\nu}\partial_\nu\ln W^{(i)},$$

$$W^{(i)} = \frac{1}{2r}\frac{shr}{chr - cost} \tag{3.21}$$

yielding for a standard (not shifted and not rotated) solution the form

$$A^{(i)}_{ia} = e_{aik}n_k(\frac{1}{r} - cthr + \frac{shr}{chr - cost}) - \delta_{ia}\frac{sint}{chr - cost}$$

$$A^{(i)}_{4a} = n_a(\frac{1}{r} - cthr + \frac{shr}{chr - cost}) \tag{3.22}$$

At large distances solutions (3.22) behave as

$$A^{(i)}_{\mu a} \sim \frac{1}{r}, \quad F^{(i)}_{\mu\nu} \sim \frac{1}{r^2} \tag{3.23}$$

and the same is true for the sum (3.1).

Hence the total action of a gas of dyons of finite length is finite. The interaction energy of the dd system, V_{dd}, and of the $d\bar{d}$ system, $V_{d\bar{d}}$ in the gauge (3.22) depends on distance r and the relative time phase φ. At large r both V_{dd} and $V_{d\bar{d}}$ fall off as $1/r$. This is shown in Figs.2 and 3. To ensure that dyonic gas could be a realistic model of the QCD vacuum one must investigate the following points:

1) to check that the classical interaction between (anti) dyons is weak enough at large distances, so that the dilute gas approximation could be reasonably justified.

2) to prove the existence of the thermodynamic limit for the dyonic ensemble (3.1), i.e. that the total action of the (big) volume V_4 is proportional to the volume, when it increases.

One must also prove that the free energy of the $d\bar{d}$ calculated with quantum corrections has a minimum at a finite (and dilute) density.

4 Confinement due to dyons

We consider first the case of one dyon and calculate its contribution to the Wilson loop in two ways:
i) first we use the long range part of the dyon field, appropriate for large Wilson loops as compared with the dyon radius ii) we show that Wilson loop can be rigorously computed through the function W of the 'tHooft ansatz

and find that the (magnetic) flux through the Wilson loop is π for the dyon and 2π for (multi)instanton; this explains why confinement is present for the first case and absent in the second in the simple picture of stochastic confinement [24].

Finally we evaluate the Wilson loop for the $d\bar{d}$ gas.

i) Consider the circular Wilson loop as in Fig.4 and the dyon at the distance h above the plane of the Wilson loop (to be the (12) plane). We are using for a large loop of radius R the quasiabelian gauge form (2.17), which enables us to exploit the Stokes theorem

$$W(C_R) = expig \int F_{12} d\sigma_{12} = exp(-i\frac{\tau_3}{2}\psi) \tag{4.1}$$

where ψ is the solid angle for the geometry of Fig.4,

$$\psi = 2\pi h \int_0^R \frac{\rho d\rho}{(\rho^2 + h^2)^{3/2}} = 2\pi(1 - \frac{h}{\sqrt{h^2 + R^2}}) \tag{4.2}$$

One can see from (4.1)-(4.2) that for large $R \gg h$ (also $R \gg b$ is necessary to use (2.17)) the color magnetic flux through the Wilson loop is

$$flux(C) = \frac{\psi}{2} = \pi \tag{4.3}$$

As we shall discuss later, this is the condition for confinement in the dyonic gas, in the picture of stochastic confinement [24].

ii) Consider now the general 'tHooft ansatz (2.1)-(2.3) and the same Wilson loop, Fig. 4, where for simplicity we put $h \ll R, h \to 0$. One has

$$W(C_R) = exp\ ig \int_{C_R} A_i dx_i = exp(\frac{i\tau_3}{2} 2\pi \frac{RW_r}{W}) \tag{4.4}$$

where W is given in (2.3), and W_r is derivate of W in $r = \sqrt{x^2}$ at $r = R$.

Two cases are possible; a) for the "periodic instanton of Harrington–Shepard [11], of the size ρ, when $R \gg \rho$, one obtains

$$W(C_R) \approx exp(2i\pi\tau_3 \frac{\rho^2}{R^2}), \tag{4.5}$$

hence flux tends to zero (or 2π) and no confinement results. The same is true for one instanton [25], in the opposite case, $\rho \gg R$, again two possibilities appear, depending on the total lengh of the instanton chain L, $L = N_1 b$.

Indeed, for $R \gg L$, one has

$$R \overset{\lim}{\to} \infty \frac{RW_r}{W} = -2 \tag{4.6}$$

and the flux is (-2π) - no confinement.

Finally for the infinite instanton chain, $L \to \infty$,

$$W(R, x_4 = 0) = \frac{\gamma}{2R} \frac{sh\gamma R}{(ch\gamma R - 1)} \to \frac{\gamma}{2R} \tag{4.7}$$

and

$$W(C_R) = \exp(-i\tau_3 \pi) = -1 \tag{4.8}$$

This coincides with the result (4.1) for $h \ll R$. Thus the dyon creates magnetic flux $(-\pi)$, while (multi)instanton creates magnetic flux (-2π). Consider now the two–dimensional gas of d and $\bar{d}$ with $2d$ density $\frac{\bar{n}}{s}$ and with Poisson distribution $w(n)$

$$w(n) = e^{-\bar{n}} \frac{(\bar{n})^n}{n!} \tag{4.9}$$

The averaged Wilson loop can be written as

$$< W(C_R) >= \sum_n e^{-\bar{n}} \frac{(-1)^n (\bar{n})^n}{n!} = e^{-2\bar{n}} = e^{-\sigma S} \tag{4.10}$$

where the string tension σ is : $\sigma = 2\frac{\bar{n}}{S}$.

Thus the $2d$ Poisson gas of dyons (and/or antidyons yields confinement with string tension proportional to the $(2d)$ average density of dyons.

Next we consider the $3d$ gas dyons, which effectively means that in the total $4d$ ansatz (3.1) we keep for simplicity all dyons with roughly the same orientation $\omega^{(i)} \equiv (0, 0, 0, 1)$. We again can use the quasiabelian gauge (2.17) and write for the dilute gas with N dyons inside volume V_3

$$< W(C_R) >=< exp(-\frac{i\tau_3}{2} \sum_{i=1}^{N} \psi^i) >=< cos\frac{\bar{\psi}}{2} >^N, \tag{4.11}$$

where

$$< cos\frac{\bar{\psi}}{2} >= \int \frac{d^3 r}{V_3} < cos\frac{\psi(r)}{2}. \tag{4.12}$$

Now for large volume V_3, $V_3 \gg R^3$, one has

$$< W(C_R) >= exp(-\sigma S) \tag{4.13}$$

where we have defined

$$\sigma = -\frac{N}{S} ln < cos\frac{\bar{\psi}}{2} > \tag{4.14}$$

For dyons distant from the Wilson loop one can write

$$\psi(r) = \frac{\pi R^2}{r^2} cos\theta, \tag{4.15}$$

where θ is the angle beween the direction to the dyon (vector $\vec{r}$) and the perpendicular to the Wilson loop plane, R is the radius of the Wilson loop. Expanding in (4.14) for small $\psi^2 \ll 1$ one has

$$< \psi^2 >\sim \frac{R^3}{V_3}; \sigma \approx C\frac{N}{V_3}R \tag{4.16}$$

In (4.16) c is a numerical constant, N–total number of dyons in the volume V_3. Appearance of R in (4.16) actually violates the area law eq. (4.13). Indeed, σ in (4.16) grows with the radius of the Wilson loop indefinitely; this situation may be called the superconfinement.

One should note however, that the superconfinement occurs for the ideal gas of d and $\bar{d}$, when one neglects completely correlations between d and $\bar{d}$. In reality however for the tightly correlated pair $d\bar{d}$ the long range field disappears. Indeed, adding the vector potential (3.22) for the dyon and that of $\bar{d}$, which obtains by changing the sign of $A^{(i)}_{4a}$ and the second term of $A^{(i)}_{ia}$ in (3.22) one has

$$A_{ia}(d\bar{d}) = 2e_{aik}n_k(\frac{1}{r} - coth\ r + \frac{sin\ hr}{cos\ hr - cost}) \tag{4.17}$$

$$A_{4a}(d\bar{d}) = 0$$

It is easy to calculate that the long range color magnetic and color electric fields for vector potentials (4.17) disappear,

$$E_{ia}(d\bar{d}) = 0, \quad B_{ia}(d\bar{d}) = 0(e^{-r}) \tag{4.18}$$

Hence for a given dyon, antidyons can partly screen its field and vice versa, and the phenomenon of Debye screening must take place. One can estimate the Debye screening mass for the $4d$ gas of dyons and antidyons [23]

$$m_D^2 \sim (\frac{N}{V_3})^{2/3}, \quad r_D = \frac{1}{m_D} \tag{4.19}$$

so that the Debye radius is of the order of average distance between neighboring dyons. This means that dyons which are farther away from the Wilson loop than r_D do not participate in the creation of string tension and one should replace in (4.16) R by r_D for $R \gg r_D$. Finally one obtains an estimate for Debye screened $d\bar{d}$ gas

$$\sigma \approx const m_D^2 \sim (\frac{N}{V_3})^{2/3} \tag{4.20}$$

This behaviour of σ ensures the area law for the Wilson loop, the superconfinement due to the Debye screening transforms into the confinement.

These estimates have been checked in [23] by numerical calculations of the Wilson loop.

In Figs. 5,6 the contribution to the Wilson loop from a dyon or a tight $d\bar{d}$ pair is shown as a functions of position of d or $d\bar{d}$. One can see that the dyon contribution is equal to π when dyon is inside the Wilson loop, and the $d\bar{d}$ pair contributes only when it is exactly on the Wilson contour.

5 Chiral symmetry breaking in the dyonic gas

We follow in this chapter the recent paper [14]. For the gas made of equal number of dyons N_+ and antidyons N_-, $N_+ = N_- = \frac{N}{2}$ in the big volume V_4 we assume that the thermodynamic limit exists for the total action and other extensive quantities like the free energy, when $N \to \infty$, $V_4 \to \infty$ and $\frac{N}{V_4}$ is fixed and finite.

We shall use for the dyonic gas the formalism similar to that exploited for the instanton gas by Diakonov and Petrov [5]. To study the CSB as manifested in the nonzero chiral quark mass and chiral condensate it is enough in case of instanton gas to consider only the case of one flavour, $N_f = 1$,

since the so–called consistency condition displaying CSB comes out the same also for $N_f = 2, 3$ [5,16]. Therefore we for simplicity confine ourselves in this section also to the case $N_f = 1$.

The main driving mechanism for CSB is provided by the zero fermionic modes on the topological charge [6,5]. For the instanton case zero fermionic modes were found by 'tHooft [26], and later in [5] those have been used to demonstrate CSB in the dilute instantonic gas.

In case of dyons fermionic modes have been found in [27]. For our purposes we consider two sets (They can be expressed one through another) of zero modes, one with continuous parameter β playing the role of quasimomentum.

$$\psi^{(\beta)} = W^{1/2}(\partial_0 + i\partial_i\sigma_i)(W^{-1}F^{(\beta)})U_+ \tag{5.1}$$

where $x_0 \equiv x_4$,

$$F^{(\beta)} = \sum_{n=-\infty}^{\infty} \frac{e^{i\beta n 2\pi}}{r^2 + (x_0 + 2\pi n)^2}, \tag{5.2}$$

and U_+ is a constant spinor of positive chirality.

Another set is labbeled by the integer n and is obtained from $\psi^{(\beta)}$ putting $\beta = 0$ and keeping only one term in the sum over n.

$$u_n(x) = W^{1/2}(\partial_0 + i\partial_i\sigma_i)(W^{-1}\frac{U_+}{r^2 + (x_0 + 2\pi n)^2}) \tag{5.3}$$

In what follows we shall use both sets.

The main problem to be solved in this section is: given fermionic zero modes on each of dyons and antidyons; find the full quark Green's function for the dyonic gas with the superposition ansatz (3.1).

To this end we make the same interpolating approximation for the one-dyon quark Green's function $S^{(i)}$ as in [5], i.e. in the exact spectral representation of $S^{(i)}$, $i = 1, \ ...N$

$$S^{(i)}(x, y) = \sum_n \frac{u_n^{(i)}(x)u_n^{(i)+}(y)}{\lambda_n - im} \tag{5.4}$$

containing all modes $n = 1, 2, ...\infty$, we keep only zero modes $u_s^{(i)}$ and replace the nonzero-mode contribution by the free Green's function, since they coincide at large $n \sim \sqrt{p^2}$.

Thus with $S_0 = (-i\hat{D}(B) - im)^{-1}$ one has

$$S^{(i)}(x,y) = S_0(x,y) + \sum_{\text{zero modes}} \frac{u_s^{(i)}(x) u_s^{(i)+}(y)}{-im} \tag{5.5}$$

One can see that $S^{(i)}$ diverges as $m \to 0$, we shall show however that the total Green's function is finite for $m \to 0$ if $N_+ = N_-$.

Using (5.5) and (31) one derives the total Green's function to be

$$S = S_0 - \sum_{\substack{i,k \\ n,m}} u_n^{(i)}(x) \left(\frac{1}{im + \hat{V}} \right)_{\substack{ik \\ nm}} u_m^{(k)+}(y) \tag{5.6}$$

where upper indices i, k run over all dyon numbers, $1 \le i,\ k \le N$, while lower indices n, m run over all set of zero modes of the given dyon with the numbers i, k. We have also defined

$$V_{nm}^{ik} \equiv \int u_n^{(i)+}(x) i(\hat{\partial} - ig\hat{B}) u_m^{(k)}(x) d^4x \tag{5.7}$$

We keep here the field B to make the formalism gauge invariant; in estimates we systematically put B_μ equal to zero. Note that u_n^+ and u_m in (6.4) should have opposite chiralities, hence V^{ik} refer to dyon–antidyon ($d\bar{d}$) or opposite ($\bar{d}d$) transitions, otherwise $V^{ik}$is zero.

One can introduce graphs as in [5] to describe each term in (5.6) as a propagation amplitude from a dyon i to a dyon k through scattering on many intermediate (anti)dyons centers, with scattering amplitude of each center (dyon) being $\frac{1}{im}$ and transition amplitude from center j (excited to the s-th level) to center l (excited to the r-th level) being V_{sr}^{jl}.

The lower indices are not the only new element in (5.5) as compared to the instanton gas model [5,16]. The zero modes $u_n^{(i)}$ depend also on the Lorentz orientation $\omega^{(i)}$ of dyon, in addition to the color orientation $\Omega^{(i)}$ and position $R^{(i)}$ of the dyon, see Eq. (3.2).

$$u_n^{(i)}(x) = \Omega^{(i)} u_n^{(i)}(x - R^{(i)}, \omega^{(i)}) \tag{5.8}$$

Our next task is to compute the matrix elements of $(\frac{1}{im+\hat{V}})_{\substack{ik \\ nm}}$ fixing initial and final states and averaging over all coordinates of intermediate dyons. To

this end we introduce as in [5] the amplitudes D^{ik}_{nm} and P^{ik}_{nm} for even and odd number of transitions $\hat{V}$ respectively

$$(\frac{1}{im+\hat{V}})_{\substack{ik\\nm}} = \frac{\delta_{ik}\delta_{nm}}{im} + \left\{ \begin{array}{l} D^{ik}_{nm}(R^{(i)}_i, R^{(k)}, \Omega^{(i)}, \Omega^{(k)}, \omega^{(i)}, \omega^{(k)}) \\ P^{ik}_{nm}(R^{(i)}_i, R^{(k)}, \Omega^{(i)}, \Omega^{(k)}, \omega^{(i)}, \omega^{(k)}) \end{array} \right. \tag{5.9}$$

In the definition (5.9) it is assumed that amplitudes of returns to the initial and final center i ar k are not included in D^{ik}, P^{ik} and should be added separately (which makes Eq.(5.9) not an equality, but rather a symbolic equation). This amplitude of the return to the center j we denote as

$$\Delta_{mn} = D^{jj}_{mn}(R^{(j)}, R^{(j)}, \Omega^{(j)}, \Omega^{(j)}, \omega^{(j)}, \omega^{(j)}) \tag{5.10}$$

Since in D^{jj} integration over all intermediate coordinates $(R^{(k)}, \Omega^{(k)}, \omega^{(k)})$ is done, Δ_{mn} does not depend on $R^{(j)}, \Omega^{(j)}, \omega^{(j)}$ and is a constant matrix.

Taking into account any number of returns to the same center j, brings about a matrix ε_{mn}, defined as:

$$\varepsilon_{mn} = \frac{1}{m}(1 - im\hat{\Delta})^{-1}_{mn} \tag{5.11}$$

With its help the equations, connecting $\hat{P}$ and $\hat{D}$ can be written as follows

$$P^{ik}_{nm} = -\frac{1}{im}V^{ik}_{nm}\frac{1}{im} - \frac{N}{2V_4}\int d^4R^{(j)}d\Omega^{(j)}d\omega^{(j)}\frac{1}{i}V^{ij}_{ns}\varepsilon_{sm'}D^{jk}_{m'm} \tag{5.12}$$

$$D^{ik}_{nm} = -\frac{N}{2V_4}\int d^4R^{(j)}d\Omega^{(j)}d\omega^{(j)}\frac{1}{i}V^{ij}_{ns'}\varepsilon_{s's}P^{jk}_{sm} \tag{5.13}$$

As a next step we separate out the dependence of $\hat{P}, \hat{D}, \hat{V}$ on lower indices and on $\Omega^{(i)}, \Omega^{(k)}$. To this end we consider zero-mode solutions $u^{(i)}_n(x)$ in the form of (5.3) and make Fourier transform

$$u^{(i)}_n(p) = \int u^{(i)}_n(x)e^{ipx}d^4x = e^{iP_0 2\pi n}\bar{u}^{(i)}(p) \tag{5.14}$$

It is important that $\bar{u}^{(i)}(p)$ does not depend on n altogether. Therefore with the help of (5.7) one has

$$V^{ij}_{nm}(R^{(i)}, \Omega^{(i)}, \omega^{(i)}; R^{(j)}, \Omega^{(j)}, \omega^{(j)}) = \int \frac{d^4p}{(2\pi)^4}e^{ip(R^{(i)}-R^{(j)})}v^{ij}_{nm}(p), \tag{5.15}$$

$$v^{ij}_{nm}(p) = e^{-2\pi i(p^i_0 - p^j_0 m)}\bar{u}^+(p^i)\Omega^{+(i)}(-\hat{p})\Omega^{(j)}\bar{u}(p^j) \tag{5.16}$$

where $p^i = \Re_{\omega_i} p$, and $\Re_{\omega_i}$ is 0(4) rotation transforming time unit vector into ω_i.

We introduce now "amputated" amplitudes d, f, w instead of $\hat{D}, \hat{P}, \hat{V}$ as follows

$$D^{ik}_{mn} = \int \frac{d^4p}{(2\pi)^4} e^{ip(R^{(i)}-R^{(k)})-i2\pi(p^i_0 m - p^k_0 n)}\bar{u}^+(p^i)\Omega^{+(i)}d(p^i,p^k)\Omega^{(k)}\bar{u}(p^k) \tag{5.17}$$

and similarly for $f(p^i,p^k)$; according to (5.16)) one has $w(p^i,p^k) \equiv -\hat{p}$

Insertion of these definitions into Eqs.(5.12-5.13) yields

$$f(p^i,p^k) = -\frac{w(p)}{(im)^2} - \frac{N}{2V_4N_c}\frac{w}{i}\int \nu(p^j)d\omega^{(j)}d(p^j,p^k) \tag{5.18}$$

$$d(p^i,p^k) = -\frac{N}{2V_4N_c i}w(p)\nu(p^j)d\omega^{(j)}f(p^j,p^k) \tag{5.19}$$

where we have introduced

$$\nu(p) = \sum_{n,s} e^{+ip_0 2\pi n}\bar{u}(p)\varepsilon_{ns}e^{-ip_0 2\pi s}\bar{u}^+(p) \tag{5.20}$$

One can see in (5.18-5.19) that f and d do not depend on rotations in p^i, p^k and the integration over $d\omega^i$ there acts only on $\nu(p^j)$, so that with the definition

$$\bar{\nu}(p) = \int \nu(p_j)d\omega^j \tag{5.21}$$

one obtains

$$d(p) = \frac{\frac{iN\hat{p}\bar{\nu}\hat{p}}{2V_4N_c m^2}}{1+\left(\frac{N}{2V_4N_c}\right)^2\hat{p}\bar{\nu}\hat{p}\bar{\nu}} \tag{5.22}$$

and $f(p)$ is expressed through d via (5.18). The definition (5.10) can be used now to obtain the selfconsistency relation, taking into account that at $m \to 0$, $\tilde{\Delta} \sim \frac{1}{m^2}$ and therefore one has

$$\Delta_{mn}\varepsilon_{ns} = \frac{i}{m^2}\delta_{ms} \tag{5.23}$$

as a result of insertion of (5.17) and (5.22) into (5.10) multiplied with ε_{mn}, one has

$$n_0 = \frac{2V_4 N_c}{N} \int \frac{d^4p}{(2\pi)^4} \frac{M^2(p)}{M^2(p) + p^2} \tag{5.24}$$

where we have defined the average number of zero modes per dyon $-n_0$, $n_0 \approx \frac{V_4^{1/4}}{b}$, b is the internal scale parameter of dyons,

We also introduced the chiral mass $M(p)$

$$M(p) = \frac{N}{2V_4 N_c} tr(\hat{p}\bar{\nu}(p)\hat{p}) = \frac{N}{2V_4 N_c} p^2 \bar{\nu}(p) \tag{5.25}$$

where we used the fact that $\bar{\nu}$ is averaged over all directions and should be proportional to the unit matrix in Lorentz and color space.

Eq.(5.24) goes over into the corresponding consistency relation for instantons [5.16] when $n_0 = 1$ and matrix $\hat{\varepsilon}$ becomes a number, while $\bar{u}(p)$ is the Fourier transform of the 'tHooft's zero mode [26].

The solution $d(p)$ (5.22) assumes the knowledge of the matrix ε_{ns}, while the consistency relation (5.24) imposes only one condition. Therefore the strategy of solution is as follows. From (5.17) one finds $\Delta_{mn} \equiv D^{ii}_{mn} = \int \frac{d^4p}{(2\pi)^4} e^{-2\pi i p_0 (m-n)} \bar{u}^+(p) d(p) \bar{u}(p)$ through $d(p)$. It clearly depends only on the modulus $|m-n|$. Then inverting the matrix Δ_{mn} one finds ε_{mn} from (5.23). Finally from (5.20-5.21) one finds $\bar{\nu}(p)$ and inserts it into (5.22), defining $d(p)$ The cycle is thus completed, and should be repeated till the convergence is achieved.

One can also study another basis of zero modes, namely that of (5.1).

In this case dependence on β can be also extracted, indeed

$$u^{(i)}_\beta(p) = \sum_n e^{-i\beta 2\pi n} u^{(i)}_n = \sum_n e^{2\pi n i (p_0 - \beta)} \bar{u}^{(i)}(p) =$$

$$= \sum_k \delta(\beta - p_0 - k) \bar{u}^{(i)}(p) \equiv \delta_{[\beta, p_0]} \bar{u}^{(i)}(p) \tag{5.26}$$

where we have introduced notation $\delta_{[\beta p_0]}$, implying that β is in the interval [0,1] and δ – function should be moderated first, introducing finite number of centers N_0 in the dyon ($\sum_{n=-N_0/2}^{N_0/2}$) and considering limit $N_0 \to \infty$ at the end.

In this way one obtains the same equations (5.17-5.19) for f, d if the new definitions are used, e.g.

$$D^{ik}_{\beta\beta'} = \int \frac{d^4 p e^{ip(R^i - R^k)}}{(2\pi)^4} \delta_{[\beta p_0]} \bar{u}^+(p^i) \Omega^{+i} d(p^i p^+) \Omega^k \bar{u}(p^k) \delta_{[\beta' p_0]} \tag{5.27}$$

and where in (5.18-5.19) now $\bar{\nu}$ is defined as

$$\bar{\nu}(p) \to \tilde{\nu}(p) = \int d\omega \bar{u}(p) \varepsilon_{[p_0, p_0]} \bar{u}^+(p) \tag{5.28}$$

and

$$\varepsilon_{[p_0, p_0]} \equiv \int_0^1 d\beta \int_0^1 d\beta' \delta_{[\beta, p_0]} \varepsilon_{\beta\beta'} \delta_{[\beta', p_0]} \tag{5.29}$$

From (5.27) one deduces that $\Delta_{\beta\beta'} = \delta_{\beta\beta'} \Delta(\beta)$ and hence also $\varepsilon_{\beta\beta'}$ is diagonal due to the relation

$$\varepsilon_{\beta\beta'} \Delta_{\beta'\beta''} = \frac{i}{m^2} \delta_{\beta\beta''} \tag{5.30}$$

and is equal to

$$\varepsilon_{\beta\beta'} = \delta_{\beta\beta'} \varepsilon(\beta) = \delta_{\beta\beta'} \frac{i}{m} \Delta^{-1}(\beta) \tag{5.31}$$

with

$$\Delta(\beta) = \int \frac{d^4 p}{(2\pi)^4} \delta_{[\beta p_0]} \bar{u}^+(p) d(p) \bar{u}(p) \tag{5.32}$$

The system (5.22), (5.24-5.29) is now complete.

We now proceed to write down the quark propagator (5.6) in terms of functions d, f and finally in terms of the chiral mass $M(p)$ (5.25).

Following the same procedure as in [5], one can rewrite (5.6) as

$$S(p) = \frac{\hat{p}}{p^2} - \frac{N}{2V_4} \left(\frac{\delta_{ns}}{im} + \left(\Delta \frac{1}{1 - im\Delta} \right)_{ns} \right) \times$$

$$\times \int d\Omega^{(i)} d\omega^{(i)} (u_n^{(i)}(p, \omega^{(i)}) u_s^{(i)+}(p, \omega^{(i)}) + \text{ dyon } \leftrightarrow \text{ antidyon}) -$$

$$- \left(\frac{N}{2V_4} \right)^2 \int d\Omega^{(i)} d\Omega^{(j)} d\omega^{(i)} d\omega^{(j)} u_n^{(i)}(p, \omega^{(i)}) (m\varepsilon_{ns}) D_{sl}^{ij} (m\varepsilon_{lk}) u_k^{+j}(p, \omega^j)$$

$$- \left(\frac{N}{2V_4} \right)^2 \int d\Omega^{(i)} d\Omega^{(j)} d\omega^{(i)} d\omega^{(j)} u_n^{(\bar{i})}(p, \omega^{(i)}) (m\varepsilon_{ns}) D_{sl}^{\bar{i}\bar{j}} (m\varepsilon_{lk}) u_k^{+\bar{j}}(p, \omega^j)$$

$$-\left(\frac{N}{2V_4}\right)^2\int d\Omega^{(i)}d\Omega^{(j)}d\omega^{(i)}d\omega^{(j)}[u_n^{(i)}(p,\omega^{(i)})(m)\varepsilon_{ns}P_{sl}^{i\bar{j}}(m)\varepsilon_{lk}u_k^{+\bar{j}}(p,\omega^j)$$

$$+(i\to\bar{i},\bar{j}\to j)] \tag{5.33}$$

Submitting in (5.33) expression (5.17-5.19) and (5.22), (5.24) we finally obtain $S(p)$ in the form

$$S(p)=\frac{\hat{p}+iM(p)}{p^2+M^2} \tag{5.34}$$

This form justifies the meaning of $M(p)$ as a chiral mass, i.e. an effective mass of quark due to CSB. It coincides with the form of $S(p)$ for the instantonn gas [5], however the explicit expression for $M(p)$ (5.25) differs.

The most remarkable feature of (5.34) is the disappearance of the massless pole $\frac{\hat{p}}{p^2}$ from $S_0(p)$ One should have in mind of course that the form (5.34) is gauge–noninvariant and obtained neglecting confinement. If one takes into account these effects, as in [27], the pole structure in (5.34) is supplemented by the area law due to the string between the given quark and an antiquark and the pole is never present in physical amplitudes.

From (5.34) one can easily compute the chiral condensate:

$$<\bar{q}q>_{Mink.}=-i<\bar{q}q>_{Eucl.}=i<trS(x,x)>=$$

$$=i\int\frac{d^4p}{(2\pi)^4}S(p)=-4N_c\int\frac{d^4p}{(2\pi)^4}\frac{M(p)}{p^2+M^2(p)} \tag{5.35}$$

It is nonzero thus confirming the phenomenon of CSB in the dilute dyonic gas.

6 Conclusions and prospectives

We have given arguments that the dilute dyonic gas provides confinement and CSB and therefore may be a good candidate for a realistic quasiclassical QCD vacuum. Additional numerical checks are necessary of the area law of the Wilson loop for the 0(4) invariant $4d$ dyonic gas, which are now in progress [23]. If confirmed, the dyonic gas ansatz can be used in the same program of detailed calculations as were done for the instanton gas [8]. In addition one can calculate field correlators and condencates to be used as input in OPE and the vacuum correlator method [3,22,27].

Meanwhile in anticipation of exact numerical checks let us estimate roughly parameters of dyonic gas which could ensure realistic values of 1) gluonic condensate 2) chiral condensate 3) string tension 4) topological susceptibility. To get 1) and 4) at realistic values one needs roughly density of one topological charge per $1fm^4$. This can be saturated by $3d$ density of dyons of 1 dyon per $1fm^3$ and with $b \approx 1fm$. Then the average size of dyon is $\gamma^{-1} = \frac{b}{2\pi} \approx 0.16fm$ and one expects from Eqs. (5.24), (5.25) and (5.35) to get a realistic (within a factor of 2-3) chiral condensate. Finally, with the given dyon density the string tension (4.20) will be of a reasonable order of magnitude, $\sigma \sim$ several units $\times fm^{-2}$. Thus order of magnitude estimates show that realistic model of dyonic gas is feasible.

As a last point in this lecture we discuss now a possible scenario of temperature phase transition in the dyonic gas vacuum.

The confined phase is described by the gas of dyons – better to say, gas of dyonic lines which are oriented in all directions; at $T = 0$ these directions ($\omega_\mu^{(i)}$, see Eq. (3.2)) are spread uninformly in the 0(4), but at $T > 0$ the distribution of $\omega_\mu^{(i)}$ may be deformed. It is important, that d and $\bar{d}$ are not paired, i.e. the correlation length of a $d\bar{d}$ pair is of the order of average distance between D and $\bar{d}$, or $n^{-1/3}$.

The deconfined phase can be chosen in such a way, that all dyons with lines directed along axis 1,2,3 are paired, i.e. d and $\bar{d}$ form neutral $d\bar{d}$ atoms with average size of γ^{-1}; producing no long – range field. Therefore string tension in the planes (14), (24) and (34) vanishes – there is no confinement in the usual sense.

However, dyons and antidyons with lines $\omega_\mu^{(i)}$ along the 4-th axis are not paired, the $d\bar{d}$ average distance is of the order of $n^{-1/3}$ and the confinement in the spacial planes (1,2), (1,3) and (2,3) persists. One can distinguish between two phases. If in the confining phase one can write

$$S_{tot}^{conf} = \sum_{\nu=1}^{4} (S_\nu(d) + S_\nu(\bar{d})) = 8S_0, \quad S_0 = \frac{8\pi^2}{g^2}\frac{L}{b}$$

then assuming that the number of d and $\bar{d}$ does not change, in the deconfined phase one obtains

$$S_{tot}^{deconf} = \sum_{\nu=1}^{3} S_\nu(d\bar{d}) + S_4(d) + S_4(\bar{d}) \cong 5.3S_0,$$

where we have put $S_\nu(d\bar{d}) \approx 1.1.S_0$ in agreement with the estimate (3.19). Thus one can see that the gluon condensate changes by some 40% across the phase transition; this fact roughly agrees with the magnetic confinement model of ref. [3], yielding reasonable estimates of T_c (see lecture "Hot non-perturbative QCD" by the same author).

One can see that the dyonic gas model may explain the deconfinement transition in a sensible way, however exact numerical computations are necessary to elaborate the detailed picture.

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References

[1] Karsch F., in *QCD: 20 years later*, eds. P.M.Zerwas and H.A.Kastrup (World Scientific, Singapore) 1993

[2] Borgs C., *Nucl. Phys.* **B261** (1985) 455
Manousakis E. and Polonyi J., *Phys. Rev. Lett.* **58** (1987) 847

[3] Simonov Yu.A., *JETP Lett.* **55** (1992) 605; *Yad. Fiz.* **58** (1995) 357; hep-ph 9311216

[4] Shifman M.A., Vainshtein A.I. and Zakharov V.I. *Nucl. Phys.* **B147** (1979) 385, 448

[5] Diakonov D.I. and Petrov V.Yu., *Nucl. Phys.* **272** (1986) 457

[6] Callan C.G., Dashen R. and Gross D.J., *Phys. Rev.* **D17**, (1978) 2717
Shuryak E.V., *Nucl. Phys.* **B 203** (1982) 93, 116, 140
Diakonov D.I. and Petrov V.Yu., *Nucl. Phys.* **245** (1984) 259

[7] Callan C.G., Dashen R. and Gross D.J., *Phys. Lett.* **B66**, (1977) 375
Diakonov D.I., Petrov V.Yu. and P.V.Pobylitsa, *Phys. Lett.* **B 226** (1989) 471
Shuryak E.V., *Nucl. Phys.* **B 328** (1989) 85, 102

[8] Shuryak E.V., *Rev. Mod. Phys.* **65** (1993) 1
Chu M.-C et al. *Phys. Rev.* **D49** (1994) 6039

[9] Nambu Y. and Jona-Lasinio G., *Phys. Rev.* **122** (1961) 345
Klevansky S.P., *Rev. Mod. Phys.* **64** (1992) 649

[10] 'tHooft G., in *High Energy Physics*, Proc. EPS Intern. Conf., ed. A.Zichichi (Editrici Compositori, 1976)
Mandelstam S., *Phys. Rep.* **23** (1978) 245

[11] Prasad M.K. and Sommerfield C.M., *Phys. Rev. Lett.* **35** (1975) 760
Harrington B. and Shepard H., *Phys. Rev.* **D17** (1978) 2122
Rossi P., *Phys. Rep.* **86** (1982) 317

[12] Simonov Yu.A., *Sov. J. Yad. Fiz.* (1985) 557 and preprint ITEP-156 (1985) (unpublished)

[13] Smit J. and van der Sijs A., *Nucl. Phys.,* **B355** (1991) 603

[14] Gonzalez–Arroyo A. and Simonov Yu. A., FTUAM-95-16; hep-th 9506032, subm. to Nucl. Phys. B

[15] Grossman B. *Phys. Lett.* **61A** (1977) 86

[16] Diakonov D.I. and Petrov V.Yu., Leningrad preprint LINP (1986) 1153 (unpublished)
Shuryak E.V., *Nucl. Phys.* **B 302** (1988) 599
Nowak M.A., VerbaarschotJ.J.M. and Zahed I., *Nucl. Phys.* **324** (1989) 1

[17] Seiberg N. and Witten E., *Nucl. Phys.* **426** (1994) 19
Seiberg N., *Phys. Rev.* **D49** (1994) 6857

[18] Simonov Yu.A., *Yad. Fiz.* **41** (1985) 1601

[19] Jackiw R., *Rev. Mod. Phys.* **49** (1977) 681

[20] Gross D.J., Pisarski R.D. and Yaffe L.G., *Rev. Mod. Phys.* **53** (1981) 43
Chakrabarti A. and Comtet A., *Phys. Rev.* **D24** (1981) 3146 ibid. **D23** (1980) 1781
Kiselev V.G. and Selivanov K.G., it Phys. Lett **B213** (1988) 165

[21] Agasyan N.O. and Simonov Yu.A., preprint ITEP 78-94 *Int. J. Mod. Phys.* (in press)
Agasyan N.O., *Sov. J. Nucl. Phys.* (in press)

[22] Simonov Yu.A., *JETP Lett.* **57** (1993) 513; hep-ph 9311247, *Yad. Fiz.* **58** (1995) 113

[23] Martemyanov B.V., Molodtsov S.V., Simonov Yu.A. and Veselov A.I., (in preparation)

[24] Olesen P. *Nucl. Phys.* **B200** (1982) 381
Ambjorn J., Olesen P. and Peterson V. *Nucl. Phys.* **B240** (1984) 553
Olesen P. and Ambjorn J., *Nucl. Phys.* **B 170** (1980) 60, 265
Makeenko Yu.M., Polikarpov M.I. and Veselov A.I. *Phys. Lett.* **118** (1982) 133
Belova T.I., Makeenko Yu.M., Polikarpov M.I. and Veselov A.I. *Nucl. Phys.* **B230** (1984) 473

[25] Simonov Yu.A., *Sov. J. Nucl. Phys.* **50** (1989) 310

[26] 'tHooft G., *Phys. Rev.* **D 14** (1976) 3432

[27] Simonov Yu.A., *Yad. Fiz.* **57** (1994) 1491, hep-ph 94 01320

Figure captions

Fig. 1. The interaction energy, $V(r) \equiv S_{int}/T$, for a dyon and an antidyon in the static gauge, Eqs. (3.6-3.7) at the distance r *vs* r/b From Ref.[23].

Fig. 2. The interaction energy $V(r,\varphi) =< S_{int} > /T$, in units of S_0/T where S_{int} is averaged over the time period b, for the dd system as a function of distance $\Gamma_{dd}/b \equiv r$ and relative time phase φ. The logarithmic singularity at $r=0, \varphi=0$ is cut off by hand. From ref. [23].

Fig. 3. The same as in Fig.2 for the $d\bar{d}$ system. The absolute minimum of $V_{d\bar{d}}$ is at $\varphi = \pi, r = 0$ and is equal to $-1.3 S_0/T$.

Fig. 4. The Wilson loop of radius R in the (1,2)–plane and a dyon at the distance h above the plane.

Fig. 5. Contribution to the Wilson loop from the dyon placed at distance $h = zR$ above the plane of the loop and at distance $r.R$ from the center of the loop *vs* z and r. From ref. [23].

Fig. 6. Contribution to the Wilson loop from the tight $d\bar{d}$ pair. Notations are the same as in Fig.4. From ref. [23].